

Explaining answers on assessment tasks

When students explain their thinking, teachers can gain valuable information about students' understandings and misunderstandings. Examining how students explain their thinking can provide assessment information that is not available when students give only a numerical answer. Students may have a correct answer but their approach may involve errors and misunderstandings that are revealed by their explanations of their thinking. Students may have an incorrect answer but their thinking may be appropriate and even sophisticated. Consider the following grade 6 problem.

The entire middle years school is going on a field trip. There are 475 people going in all, including students, teachers, and parent volunteers. They will travel by bus. Each bus can carry no more than 30 people. Carrying over 30 people is unsafe and against the law. How many buses will be needed for the field trip? Explain how you figured out the answer.

Below are five student responses to the problem and comments about them. Four of the responses are numerically correct (or mostly correct) but the students' explanations of their thinking tell quite different stories about what students understand or do not understand. Think about each student response and accompanying comment. The benefits of asking students to explain their thinking should be clear and compelling.

Response 1.

Response 1 is numerically correct and indicates a high level of understanding of the problem and the relationship between division arithmetic and the meaning of division. The student sees the problem as making equal groups of 30, as indicated by the division work (subtracting multiples of 30) and the explanation of thinking. The student realizes that a remainder requires one more bus.

$$\begin{array}{r} 30 \overline{) 475} \\ \underline{300} \\ 175 \\ \underline{150} \\ 25 \end{array} \begin{array}{l} 10 \\ 5 \\ \hline 15 \end{array}$$

Answer to division is
15 R 25

I divided because I need to make equal groups. 16 buses are needed because one bus will only carry 25 people.

Response 2.

Response 2 is numerically correct but suggests that the student may not understand the arithmetic operation of division and perhaps not even the arithmetic operation of multiplication. The student is seeing the problem as a repeat addition of 30 to make groups of 30. This is not an empowering approach. The approach works for relatively simple whole numbers but it will be problematic for large whole numbers and decimal numbers. The student realizes that the extra 25 people require one more bus.

$$\begin{array}{r} 30 \\ 30 \\ 30 \\ \hline 90 \end{array} \quad \begin{array}{r} 30 \\ 30 \\ 30 \\ \hline 90 \end{array} \quad \begin{array}{r} 30 \\ 30 \\ 30 \\ \hline 90 \end{array} \quad \begin{array}{r} 30 \\ 30 \\ 30 \\ \hline 90 \end{array} \quad \begin{array}{r} 30 \\ 30 \\ 30 \\ \hline 90 \end{array}$$

$$90 + 90 + 90 + 90 + 90 = 450$$

$$475 - 450 = 25$$

I put 30 people in each bus, needed 15 buses.
I put 25 people in one bus.
Need $15 + 1 = 16$ buses

Response 3.

Response 3 is numerically correct even though the student does not directly say that 16 buses are needed. The student realizes that context must be considered when answering the problem. However, there is no clear indication that the student understands the meaning of division as splitting up into equal groups. The student may just have been lucky in picking an appropriate arithmetic operation to use or may have memorized how to do this type of problem.

$$\begin{array}{r} 15 \\ 30 \overline{) 475} \\ \underline{30} \\ 175 \\ \underline{150} \\ 25 \end{array}$$

You will need 15 big buses
and one mini bus.

Response 4.

Response 4 is numerically correct yet the thinking involved to get the answer is at best unclear and at worst faulty. The student seems to be mixing up rounding off and averaging. Neither of these concepts is relevant to solving the problem. Had no explanation of thinking been asked for, this student might easily have been seen as someone who understands division word problems.

$$\begin{array}{r} 15 \quad 25 \\ 30 \overline{) 475} \\ \underline{30} \\ 175 \\ \underline{150} \end{array}$$

averaged out 15 and 25
to about 15.
Answer is 16

I divided then averaged the answer
to nearest one.

Response 5.

Response 5 would be numerically incorrect without the explanation of thinking. Even so, there still is some "messiness" in the explanation. The student says "There are 450 people riding a big bus." This contradicts the '15' part of the numerical answer. Yet, the student does know what to do with the extra 25 people. The student seems to understand fraction representation of remainders (a sophisticated notion), yet does not seem to understand division as a way of getting remainders (the student uses multiplication to get the numerical answer). On the whole, there are indications of insight and of confusion or non-understanding in the response.

$$\begin{array}{r} 30 \\ \underline{4} \\ 120 \end{array} \quad \begin{array}{r} 30 \\ \underline{5} \\ 150 \end{array} \quad \begin{array}{r} 15 \\ \underline{30} \\ 450 \end{array} \quad 20 \text{ , , , , ,}$$

Answer is $15 \frac{25}{30}$ buses.

There are 450 people riding a big
bus and you'd have to put the other
25 in a van.